Planes, Products, & Distances

Monday, May 15, 2023 8:53 AM

part 2 so far:

a) planes : 3 different descriptions + intersections

b) vectors : $\times$ (cross), $\cdot$ (dot), $|\vec{v}|$ (length), & angle formulas

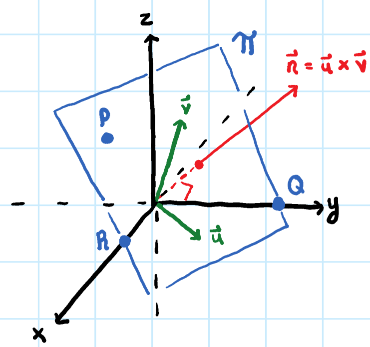
descriptions of planes:

1) π plane : P point + n orthongonal direction
   ($\perp$ to vectors on plane → $\perp$ to plane)

2) P, Q, R 3 points determine π plane

3) π plane equation:

$$\boxed{ax + by + cz = d}$$



* 4) 2 vectors in plane + point in plane *

going between descriptions:

thm : suppose π plane through point (P) & spanned by 2 vectors ($\vec{u}$ & $\vec{v}$) } description 4

then orthogonal direction is ...

$$\vec{n} = \vec{u} \times \vec{v}$$

cross product / description 1

and equation is ...

$$ax + by + cz = d$$ } description 3

where $\langle a, b, c \rangle = \vec{n} = \vec{u} \times \vec{v}$ → tells who a, b, c are

and $d = \langle a, b, c \rangle \cdot \vec{OP}$, P point in plane

$\vec{OP}$ = point P with $\langle \; \rangle$ / gives you $\langle x, y, z \rangle$ / take dot product of $\langle a, b, c \rangle \cdot \langle x, y, z \rangle = ax + by + cz$

ex) find equation for plane thru points $P = (1, 0, 0)$, $Q = (0, 2, 1)$, $R = (0, 0, -3)$

solution : choose $\vec{u} = \vec{PQ} = \langle -1, 2, 1 \rangle$
$\vec{v} = \vec{PR} = \langle -1, 0, -3 \rangle$
& P our point $= (1, 0, 0)$ } description 4

then $\vec{n} = \vec{u} \times \vec{v}$ will be orthogonal:

remember to make middle value −

$$\vec{n} = \begin{vmatrix} i & j & k \\ -1 & 2 & 1 \\ -1 & 0 & -3 \end{vmatrix} = \langle -6, -4, 2 \rangle$$

$$\vec{n} = \begin{vmatrix} i & j & k \\ -1 & 2 & 1 \\ -1 & 0 & -3 \end{vmatrix} = \langle \underset{a}{-6}, \underset{b}{-4}, \underset{c}{2} \rangle$$

remember to make middle value -

$$-6x - 4y + 2z = d \; ?$$

finally $\vec{OP} = \langle 1, 0, 0 \rangle$ & thus

$$d = \langle -6, -4, 2 \rangle \cdot \langle 1, 0, 0 \rangle$$
$$= -6 + 0 + 0$$
$$d = -6 \longrightarrow \boxed{-6x - 4y + 2z = -6}$$

check with og points